Universidad Pontificia Bolivariana
Facultad de Ingeniería Aeronáutica
Automatic Flight Control
Problem set 7
Stability, Observability, and Controllability

1. Determine if the given systems are stable or not according to the BIBO criterion.

   (a) Systems modeled by the given differential equations, where $u(t)$ is the input signal and $y(t)$ is the output signal.

      i. $(D^2 - 2D + 1)\,y(t) = (D + 1)\,u(t)$.
      ii. $(D^2 + 7D + 24)\,y(t) = (D - 1)\,u(t)$.
      iii. $(D^3 + 8D^2 + 17D + 10)\,y(t) = (D - 2)\,u(t)$.
      iv. $(D^4 + 6D^3 + 11D^2 + 6D)\,y(t) = (D^2 + 8D + 16)\,u(t)$.
      v. $(D^2 + D + 1)\,y(t) = u(t)$.
      vi. $(D^3 + 2D^2 + D + 1)\,y(t) = (D + 2)\,u(t)$.
      vii. $(D^2 + 2D + 1)\,y(t) = u(t)$.

   (b) Systems with the given transfer function, where $U(s)$ is the Laplace transform of the input signal and $Y(s)$ is the Laplace transform of the output signal.

      i. $H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{s^2 + 3s + 7}{(s+1)\,(s-3)\,(s+2)}$.
      ii. $H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{s^3 + 3s^2 + 2s + 8}{s\,(s+1)\,(s+2)\,(s+3)}$.
      iii. $H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{s\,(s^2 + 6s + 2)}{s^3 + s^2 + 5s + 1}$.
      iv. $H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{s^2 + 2s + 10}{2s^5 + s^4 + 8s^3 + 6s^2 + 2s + 12}$.
      v. $H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{s^2 - 1}{s^2 + 4s + 3}$.
      vi. $H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{2s - 3}{s^3 + 1}$.
      vii. $H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{1}{s}$.
      viii. $H(s) = \dfrac{Y(s)}{U(s)} = \dfrac{s + 4}{s^2 + 5s + 6}$.

2. Consider the linear and invariant systems modeled in state space by

$$\begin{aligned}\dot{x}(t) =& Ax(t) + Bu(t)\\ y(t) =& Cx(t) + Du(t)\end{aligned}$$

   where $x(t)$ is the state vector, $u(t)$ is the input vector and $y(t)$ is the output vector, for the following cases.

(a) $A = 1,\ B = 2,\ C = 1,\ D = 0.$

(b) $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 \end{bmatrix},\ D = \begin{bmatrix} 0 \end{bmatrix}.$

(c) $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 4 & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$

(d) $A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 \\ 9 & 0 \\ 1 & 0 \\ 2 & 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},\ D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$

(e) $A = \begin{bmatrix} -5 & 1 \\ -6 & 0 \end{bmatrix},\ B = \begin{bmatrix} 5 & 1 \\ 9 & 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix},\ D = \begin{bmatrix} -1 & 0 \\ 5 & 1 \end{bmatrix}.$

(f) $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$

For each of the given systems, do the following

i. Get the transfer matrix from the system.

ii. Determine whether the system is stable or not according to the BIBO criterion.

iii. Determine whether the system is stable or not according to the BIBS criterion.

iv. Determine whether the system is observable or not.

v. Determine whether the system is controllable or not.