Universidad Pontificia Bolivariana
Facultad de Ingeniería Aeronáutica
Automatic Flight Control
Problem set 5
Frequency Domain Analysis

1. For each of the given signals calculate the Laplace transform.

   (a) $\delta(t)$.

   (b) $1(t)$.

   (c) $r(t)$.

   (d) $p(t) = \begin{cases} \frac{t^2}{2} & t > 0, \\ 0 & t \leq 0. \end{cases}$

   (e) $e^{-at}1(t)$.

   (f) $Cos\,(w_o t)\,1(t)$.

   (g) $Sen\,(w_o t)\,1(t)$.

   (h) $e^{-at}Cos\,(w_o t)\,1(t)$.

   (i) $e^{-at}Sen\,(w_o t)\,1(t)$.

   (j) $Ae^{-at}Cos\,(w_o t + \phi)\,1(t)$.

   (k) $t\,e^{-at}1(t)$.

   (l) $t\,Cos\,(w_o t)\,1(t)$.

   (m) $t\,Sen\,(w_o t)\,1(t)$.

   (n) $t\,e^{-at}Cos\,(w_o t)\,1(t)$.

   (o) $t\,e^{-at}Sen\,(w_o t)\,1(t)$.

   (p) $At\,e^{-at}Cos\,(w_o t + \phi)\,1(t)$.

   (q) $t^2\,e^{-at}1(t)$.

   (r) $t^2\,Cos\,(w_o t)\,1(t)$.

   (s) $t^2\,Sen\,(w_o t)\,1(t)$.

   (t) $t^2\,e^{-at}Cos\,(w_o t)\,1(t)$.

   (u) $t^2\,e^{-at}Sen\,(w_o t)\,1(t)$.

   (v) $At^2\,e^{-at}Cos\,(w_o t + \phi)\,1(t)$.

   (w) $\Pi\,(t)$.

   (x) $\Lambda\,(t - 5)$.

2. Find the signals whose Laplace transforms are indicated.

   (a) $X(s) = 4\,(1 - e^{-st_o})$.

   (b) $Y(s) = \dfrac{K}{\tau s + 1}$.

(c) $Z(s) = \dfrac{Kw_o}{s + w_o}$.

(d) $Z(s) = \dfrac{Ks}{s + w_o}$.

(e) $G(s) = \dfrac{Kw_n^2}{s^2 + 2\zeta w_n s + w_n^2}$.

(f) $G(s) = \dfrac{Kw_n^2 e^{-st_d}}{s^2 + 2\zeta w_n s + w_n^2}$.

(g) $G(s) = \dfrac{2K\zeta w_n s}{s^2 + 2\zeta w_n s + w_n^2}$.

(h) $H(s) = \dfrac{Ks^2}{s^2 + 2\zeta w_n s + w_n^2}$.

(i) $Y(s) = \dfrac{s+1}{(s+2)(s+3)}$.

(j) $Y(s) = \dfrac{s^2}{(s+2)(s+3)}$.

(k) $Y(s) = \dfrac{s+1}{(s+2)^2(s+3)}$.

(l) $Y(s) = \dfrac{s-5}{(s+4)(s^2+s+1)}$.

(m) $Y(s) = \dfrac{(s^2+4s+3)e^{-3s}}{s^3+3s^2+3s+2}$.

(n) $Y(s) = \dfrac{(s^2+5s+6)}{(s+5)(s^2+s+1)^2}$.

3. Determine the initial and final values of the signals associated with the Laplace transforms of problem 2 using the initial value and final value theorems.

4. Using the Laplace transform determine the responses for each of the given systems to the input signals:

   i. $u(t) = \delta(t)$.

   ii. $u(t) = 1(t)$.

   ii. $u(t) = r(t)$.

   iv. $u(t) = e^{-t}1(t)$.

   v. $u(t) = te^{-t}1(t)$.

   vi. $u(t) = e^{-2t}1(t)$.

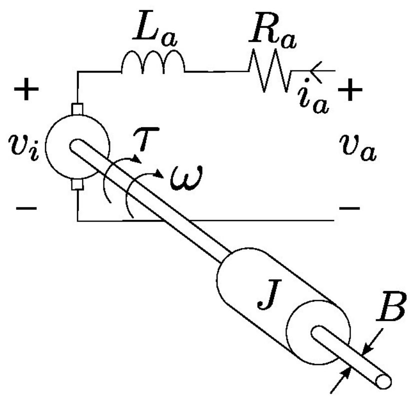
   vii. $u(t) = te^{-2t}1(t)$.

   viii. $u(t) = Sen\,(5t)\,1(t)$.

   ix. $u(t) = e^{-3t}Sen\,(5t)\,1(t)$.

   x. $u(t) = te^{-3t}Cos\,(5t)\,1(t)$.

(a) Consider a DC motor with permanent magnet as shown in the figure.

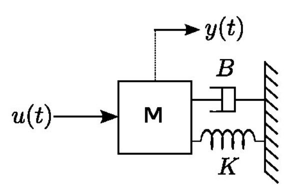


Where:

v_a : voltage across armature winding (input signal) $[=]V$.
i_a : current through armature winding $[=]A$.
R_a : armature winding resistance $[=]\Omega$.
L_a : armature winding inductance $[=]H$.
ω : angular velocity in motor axis (output signal) $[=]rad/s$.
J : moment of inertia of rotating parts of the motor $[=]kg.m^2$.
B : viscous friction coefficient of rotating parts of the motor $[=]kg.m^2/s$.
$v_i = K\omega$: voltage induced in the armature $[=]V$. K is a constant associated with the motor.
$\tau = K\eta i_a$: torque produced by the motor $[=]N \cdot m$. η is the efficiency of the motor $(0 \leq \eta \leq 1)$.

Suppose that the value of the motor parameters is: $R_a = 0.5\Omega$, $L_a = 50mH$, $J = 0.05kg \cdot m^2$, $B = 1kg \cdot m^2/s$, $K = 0.022918V \cdot s/rad$, $\eta = 0.9$.

(b) Consider the mass-spring-damper system in the figure, where $u(t)$ which is the force applied to the mass corresponds to the input signal, and $y(t)$ which is the speed of the mass corresponds to the output signal.



with $M = 1kg$, $K = 1kg/s^2$ y $B = 1kg/s$.

(c) System with transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{4}{s^2 + 2s + 1}.$$

(d) System with transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{s+2}{s^2+5s+6}.$$

(e) System with transfer function

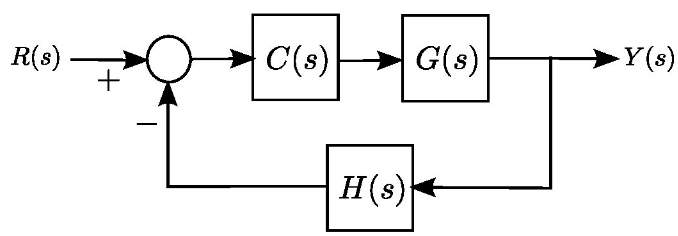
$$H(s) = \frac{Y(s)}{U(s)} = \frac{s^2+5s+4}{s^3+14s^2+59s+70}.$$

(f) System with transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{s+10}{(s+8)(s+5)}.$$

5. Check the answers to problem 4 using Octave o Matlab® and Simulink®.

6. Consider the feedback control system represented by the block diagram in the figure.



Control system error is defined as $e(t) = r(t) - y(t)$.

(a) If the transfer functions $C(s)$, $G(s)$ and $H(s)$ are given by

$$C(s) = 1 + \frac{2}{s}, \quad G(s) = \frac{10}{s+5}, \quad H(s) = \frac{1}{s+2}.$$

i. Determine the initial value of the error, $e\left(0^+\right) = \lim_{t\to 0^+} e(t)$, when the reference signal, $r(t)$, is a step.

ii. Determine the steady state error, $e_{ss} = e(\infty) = \lim_{t\to\infty} e(t)$, when the reference signal, $r(t)$, is a step.

iii. Calculate the error, $e(t)$.

(b) If the transfer functions $C(s)$, $G(s)$ and $H(s)$ are given by

$$C(s) = 1 + 2s, \quad G(s) = \frac{10}{s+5}, \quad H(s) = \frac{1}{s+2}.$$

i. Determine the initial value of the error, $e\left(0^+\right) = \lim_{t\to 0^+} e(t)$, when the reference signal, $r(t)$, is a step.

ii. Determine the steady state error, $e_{ss} = e(\infty) = \lim_{t\to\infty} e(t)$, when the reference signal, $r(t)$, is a step.

iii. Calculate the error, $e(t)$.

(c) If the transfer functions $C(s)$, $G(s)$ and $H(s)$ are given by

$$C(s) = 1 + \frac{2}{s}, \quad G(s) = \frac{10}{s+5}, \quad H(s) = 1.$$

i. Determine the initial value of the error, $e\left(0^+\right) = \lim_{t\to 0^+} e(t)$, when the reference signal, $r(t)$, is a step.
ii. Determine the steady state error, $e_{ss} = e(\infty) = \lim_{t\to\infty} e(t)$, when the reference signal, $r(t)$, is a step.
iii. Determine the initial value of the error, $e\left(0^+\right) = \lim_{t\to 0^+} e(t)$, when the reference signal, $r(t)$, is a ramp.
iv. Determine the steady state error, $e_{ss} = e(\infty) = \lim_{t\to\infty} e(t)$, when the reference signal, $r(t)$, is a ramp.
v. Calculate the error, $e(t)$.

(d) If the transfer functions $C(s)$, $G(s)$ and $H(s)$ are given by

$$C(s) = 1 + 2s, \quad G(s) = \frac{10}{s+5}, \quad H(s) = 1.$$

i. Determine the initial value of the error, $e\left(0^+\right) = \lim_{t\to 0^+} e(t)$, when the reference signal, $r(t)$, is a step.
ii. Determine the steady state error, $e_{ss} = e(\infty) = \lim_{t\to\infty} e(t)$, when the reference signal, $r(t)$, is a step.
iii. Calculate the error, $e(t)$.

7. Check the answers to problem 6 using Octave or Matlab® and Simulink®.

8. Consider a system modeled in state space by

$$\begin{aligned}\dot{\mathbf{x}}(t) =& \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) =& \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned}$$

where $\mathbf{x}(t)$ is the state vector, $\mathbf{u}(t)$ is the input vector, and $\mathbf{y}(t)$ is the output vector, with

$$\mathbf{A} = \begin{bmatrix} -5 & 1 \\ -6 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 5 & 1 \\ 9 & 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} -1 & 0 \\ 5 & 1 \end{bmatrix}.$$

For this system do the following:

(a) Obtain the state transition matrix in the frequency domain, $\mathbf{\Phi}(s)$.
(b) Get system response if $\mathbf{x}(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$ y $\mathbf{u}(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ for $t \geq 0$.
(c) Obtener la matriz de transferencia del sistema, $\mathbf{H}(s)$.
(d) Get system response if $\mathbf{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ y $\mathbf{u}(t) = \begin{bmatrix} e^{-t} & 1 \end{bmatrix}^T$ for $t \geq 0$.

(e) Get system response if $\mathbf{x}(0) = \begin{bmatrix}1 & -1\end{bmatrix}^T$ y $\mathbf{u}(t) = \begin{bmatrix}e^{-t} & 1\end{bmatrix}^T$ for $t \geq 0$.

(f) Obtain the Jordan canonical form and develop the previous literals in terms of this new representation.

9. Consider a system modeled in state space by

$$\begin{aligned}\dot{\mathbf{x}}(t) =& \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) =& \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned}$$

donde $\mathbf{x}(t)$ is the state vector, $\mathbf{u}(t)$ is the input vector, and $\mathbf{y}(t)$ is the output vector, with

$$\mathbf{A} = \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -3\end{bmatrix},\quad \mathbf{B} = \begin{bmatrix}1\\1\\1\end{bmatrix},\quad \mathbf{C} = \begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 0\end{bmatrix},\quad \mathbf{D} = \begin{bmatrix}0\\1\end{bmatrix}.$$

For this system do the following:

(a) Obtain the state transition matrix in the frequency domain, $\mathbf{\Phi}(s)$.

(b) Get system response if $\mathbf{x}(0) = \begin{bmatrix}1 & 2 & -1\end{bmatrix}^T$ y $u(t) = 0$ for $t \geq 0$.

(c) Obtain the transfer matrix for this system, $\mathbf{H}(s)$.

(d) Get system response if $\mathbf{x}(0) = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$ y $u(t) = e^{-2t}$ for $t \geq 0$.

(e) Get system response if $\mathbf{x}(0) = \begin{bmatrix}3 & 5 & -10\end{bmatrix}^T$ y $u(t) = 1$ for $t \geq 0$.

