Universidad Pontificia Bolivariana
Facultad de Ingeniería Aeronáutica
Automatic Flight Control
Problem set 4
Time Domain Analysis

1. Consider the systems modeled mathematically by:

   (a) First order system.
   $$(D+1)\,y(t) = u(t).$$

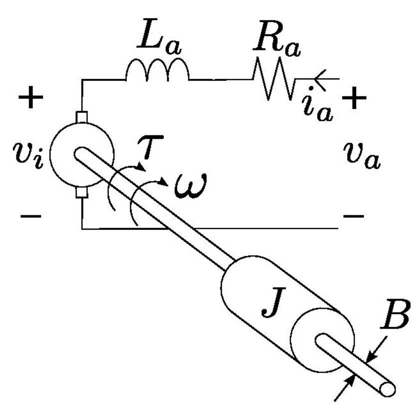
   (b) First order system high pass type.
   $$(D+1)\,y(t) = Du(t).$$

   (c) Phase lag compensator.
   $$(D+1)\,y(t) = \left(\frac{1}{10}D+1\right)u(t).$$

   (d) Phase lead compensator.
   $$\left(\frac{1}{10}D+1\right)y(t) = (D+1)\,u(t).$$

   (e) Unstable first-order system.
   $$(D-1)\,y(t) = u(t)$$

   (f) A DC motor with permanent magnet as shown in the figure.



Where:

v_a : voltage across armature winding (input signal) [=]V.
i_a : current through armature winding [=]A.
R_a : armature winding resistance [=]Ω.
L_a : armature winding inductance [=]H.
ω : angular velocity in motor axis (output signal) [=]rad/s.

J : moment of inertia of rotating parts of the motor $[=] kg.m^2$.

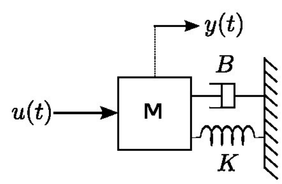
B : viscous friction coefficient of rotating parts of the motor $[=] kg.m^2/s$.

$v_i = K\omega$: voltage induced in the armature $[=] V$. K is a constant associated with the motor.

$\tau = K\eta i_a$: torque produced by the motor $[=] N \cdot m$. η is the efficiency of the motor $(0 \leq \eta \leq 1)$.

Suppose that the value of the motor parameters is: $R_a = 0.5\Omega$, $L_a = 50mH$, $J = 0.05kg \cdot m^2$, $B = 1kg \cdot m^2/s$, $K = 0.022918V \cdot s/rad$, $\eta = 0.9$.

(g) Consider the mass-spring-damper system in the figure, where $u(t)$ which is the force applied to the mass corresponds to the input signal, and $y(t)$ which is the speed of the mass corresponds to the output signal.



with $M = 1kg$, $K = 1kg/s^2$ y $B = 1kg/s$.

(h) Underdamped low-pass type second order system $(\zeta < 1)$.

$$\left(D^2 + 2\zeta w_n D + w_n^2\right) y(t) = K w_n^2 \, u(t).$$

(i) Critically damped low-pass type second order system $(\zeta = 1)$.

$$\left(D^2 + 2 w_n D + w_n^2\right) y(t) = K w_n^2 \, u(t).$$

(j) Overdamped low-pass type second order system $(\zeta > 1)$.

$$\left(D^2 + 2\zeta w_n D + w_n^2\right) y(t) = K w_n^2 \, u(t).$$

(k) Underdamped band-pass type second order system $(\zeta < 1)$.

$$\left(D^2 + 2\zeta w_n D + w_n^2\right) y(t) = K 2\zeta w_n \, D u(t).$$

(l) Critically damped band-pass type second order system $(\zeta = 1)$.

$$\left(D^2 + 2 w_n D + w_n^2\right) y(t) = K 2 w_n \, D u(t).$$

(m) Overdamped band-pass type second order system $(\zeta > 1)$.

$$\left(D^2 + 2\zeta w_n D + w_n^2\right) y(t) = K 2\zeta w_n \, D u(t).$$

(n) Underdamped high-pass type second order system $(\zeta < 1)$.

$$\left(D^2 + 2\zeta w_n D + w_n^2\right) y(t) = K D^2 \, u(t).$$

(o) Critically damped high-pass type second order system ($\zeta = 1$).

$$\left(D^2 + 2w_n D + w_n^2\right) y(t) = K D^2 \, u(t).$$

(p) Overdamped high-pass type second order system ($\zeta > 1$).

$$\left(D^2 + 2\zeta w_n D + w_n^2\right) y(t) = K D^2 \, u(t).$$

(q) $(D^2 + D + 1)\, y(t) = u(t)$.

(r) $(D^3 + 2D^2 + D + 1)\, y(t) = (D + 2)\, u(t)$.

(s) $(D^2 + 2D + 1)\, y(t) = u(t)$.

(t) $(D^2 + 4D + 3)\, y(t) = (D^2 - 1)\, u(t)$.

(u) $(D^3 + 1)\, y(t) = (2D - 3)\, u(t)$.

(v) $Dy(t) = u(t)$.

(w) $(D^2 + 5D + 6)\, y(t) = (D + 4)\, u(t)$.

For each of the listed systems do the following:

i. Find the system step response.
ii. Find the system ramp response.
iii. Find the system response for the input signal $u(t) = Cos\,(wt)\, 1(t)$.
iv. Find the system response for the input signal $u(t) = e^{-3t} 1(t)$.
v. Find the system response for the input signal $u(t) = e^{-4t} Sen\,(10t + 1)\, 1(t)$.
vi. Find the system impulse response.
vii. Draw a block diagram that represents the system in terms of integrators, adders, and multiplication by a constant. Verify the answers to the problems developed using the block diagram in Simulink ®.

It is suggested to use the following procedure:

- Determine the initial conditions from the input signal and the differential equation.
- Find the form of the natural response for which you must pose and solve the characteristic equation and write the form of the natural response from the poles of the system.
- Find the forced response:
  - If you use undetermined coefficients find the annihilator operator associated with the input; solve the obtained homogeneous differential equation and state the form of the forced response; replace in the original equation to get the undetermined coefficients and thus the forced response.
  - If you use transfer function, first find the transfer function $H(s)$; evaluate the transfer function at the value of s associated with the input; and determine the forced response according to the form of the input and the transfer function.
- Obtain the form of the total response and evaluate the initial conditions to find the coefficients of the natural response and thus obtain the total response.

2. Consider a system modeled in state space by

$$\begin{aligned}\dot{\mathbf{x}}(t) =& \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) =& \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned}$$

where $\mathbf{x}(t)$ is the state vector, $\mathbf{u}(t)$ is the input vector, and $\mathbf{y}(t)$ is the output vector, with

$$\mathbf{A} = \begin{bmatrix}-5 & 1\\ -6 & 0\end{bmatrix}, \mathbf{B} = \begin{bmatrix}5 & 1\\ 9 & 0\end{bmatrix}, \mathbf{C} = \begin{bmatrix}1 & 0\\ -5 & 1\end{bmatrix}, \mathbf{D} = \begin{bmatrix}-1 & 0\\ 5 & 1\end{bmatrix}.$$

For this system do the following:

(a) Get the state transition matrix in the time domain, $\phi(t) = \mathbf{e}^{\mathbf{A}t}$.

(b) Get system response if $\mathbf{x}(0) = \begin{bmatrix}1 & -1\end{bmatrix}^T$ y $\mathbf{u}(t) = \begin{bmatrix}0 & 0\end{bmatrix}^T$ for $t \geq 0$.

(c) Get the impulse response matrix for this system, $\mathbf{h}(t)$.

(d) Get system response if $\mathbf{x}(0) = \begin{bmatrix}0 & 0\end{bmatrix}^T$ y $\mathbf{u}(t) = \begin{bmatrix}e^{-t} & 1\end{bmatrix}^T$ for $t \geq 0$.

(e) Get system response if $\mathbf{x}(0) = \begin{bmatrix}1 & -1\end{bmatrix}^T$ y $\mathbf{u}(t) = \begin{bmatrix}e^{-t} & 1\end{bmatrix}^T$ for $t \geq 0$.

(f) Obtain the Jordan canonical form and develop the previous literals in terms of this new representation.

3. Consider a system modeled in state space by

$$\begin{aligned}\dot{\mathbf{x}}(t) =& \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) =& \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned}$$

donde $\mathbf{x}(t)$ is the state vector, $\mathbf{u}(t)$ is the input vector, and $\mathbf{y}(t)$ is the output vector, with

$$\mathbf{A} = \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -3\end{bmatrix}, \mathbf{B} = \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}, \mathbf{C} = \begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}, \mathbf{D} = \begin{bmatrix}0\\ 1\end{bmatrix}.$$

For this system do the following:

(a) Get the state transition matrix in the time domain, $\phi(t) = \mathbf{e}^{\mathbf{A}t}$.

(b) Get system response if $\mathbf{x}(0) = \begin{bmatrix}1 & 2 & -1\end{bmatrix}^T$ y $u(t) = 0$ for $t \geq 0$.

(c) Get the impulse response matrix for this system, $\mathbf{h}(t)$.

(d) Get system response if $\mathbf{x}(0) = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$ y $u(t) = e^{-2t}$ for $t \geq 0$.

(e) Get system response if $\mathbf{x}(0) = \begin{bmatrix}3 & 5 & -10\end{bmatrix}^T$ y $u(t) = 1$ for $t \geq 0$.

4. Explore the use of computational tools for the simulation of dynamic systems modeled by differential equations, transfer functions or in state space. In particular, explore the functions *tf*, *ss*, *step*, *impulse*, *linsim*, and *ode45* of the Octave and Matlab® m language.