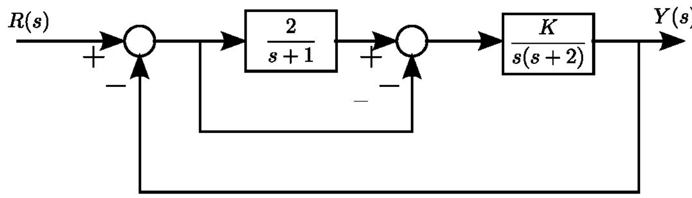
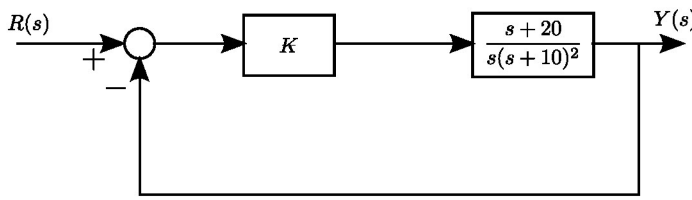
Universidad Pontificia Bolivariana
Facultad de Ingeniería Aeronáutica
Automatic Flight Control
Problem set 3
Block Diagram Models

1. Obtain the equivalent transfer function from the following block diagrams.

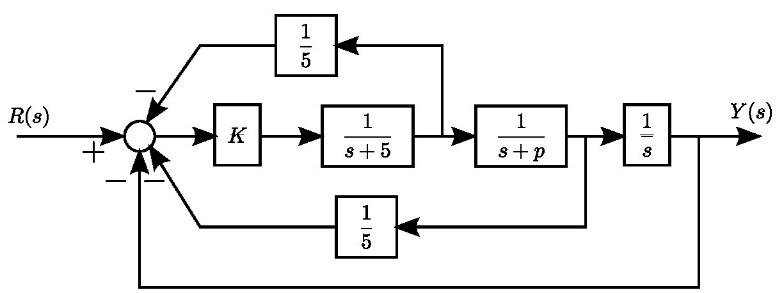
   (a)



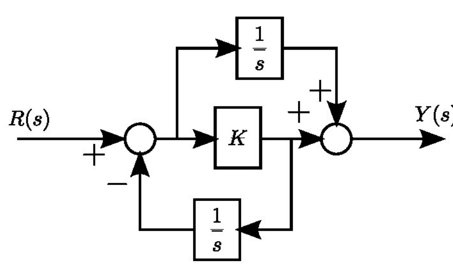
   (b)



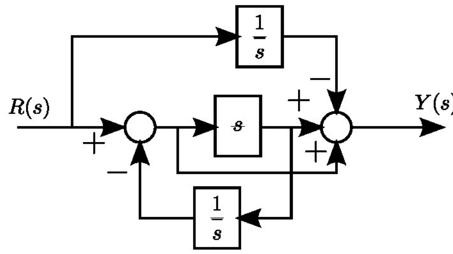
   (c)



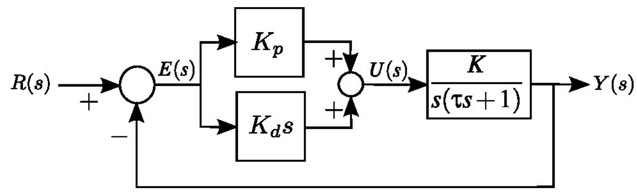
   (d)



   (e)



2. Consider the proportional derivative control system (PD) modeled by the block diagram shown in the figure.



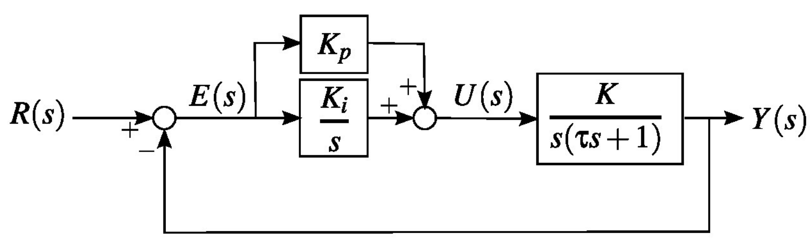
Obtain the following transfer functions for this system:

(a) $\dfrac{Y(s)}{R(s)}$.

(b) $\dfrac{U(s)}{R(s)}$.

(c) $\dfrac{E(s)}{R(s)}$.

3. Consider the proportional integral control system (PI) modeled by the block diagram shown in the figure.



Obtain the following transfer functions for this system:

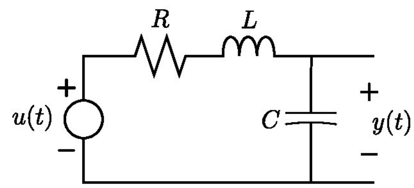
(a) $\dfrac{Y(s)}{R(s)}$.

(b) $\dfrac{U(s)}{R(s)}$.

(c) $\dfrac{E(s)}{R(s)}$.

4. Draw a block diagram in which all the internal signals of the system can be seen for each of the following systems:

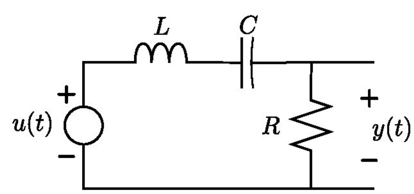
(a) Consider an RLC electrical circuit as shown in the figure.



Where:

$u(t)$: source voltage $[=]V$.
$y(t)$: output voltage (voltage across C) $[=]V$.
R : electric resistance $[=]\Omega$.
L : inductance $[=]H$.
C : capacitance $[=]F$.

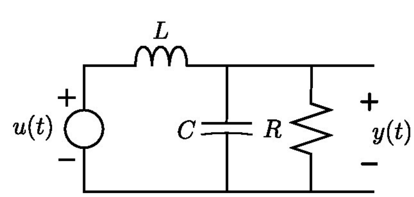
(b) Consider an LCR electrical circuit as shown in the figure.



Where:

$u(t)$: source voltage $[=]V$.
$y(t)$: output voltage (voltage across R) $[=]V$.
R : electric resistance $[=]\Omega$.
L : inductance $[=]H$.
C : capacitance $[=]F$.

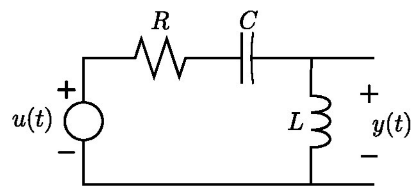
(c) Consider an LCR electrical circuit as shown in the figure.



Where:

$u(t)$: source voltage $[=]V$.
$y(t)$: output voltage (voltage across R) $[=]V$.
R : electric resistance $[=]\Omega$.
L : inductance $[=]H$.
C : capacitance $[=]F$.

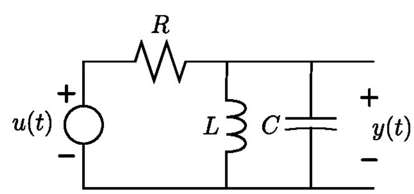
(d) Consider an RCL electrical circuit as shown in the figure.



Where:

$u(t)$: source voltage $[=]V$.
$y(t)$: output voltage (voltage across L) $[=]V$.
R : electric resistance $[=]\Omega$.
L : inductance $[=]H$.
C : capacitance $[=]F$.

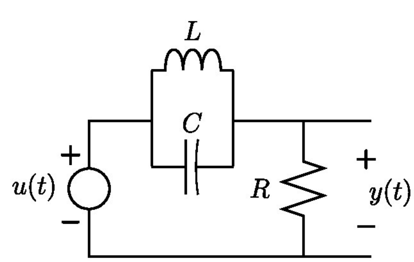
(e) Consider an electrical RLC circuit with an LC tank circuit in parallel as shown in the figure.



Where:

$u(t)$: source voltage $[=]V$.
$y(t)$: output voltage (voltage across L or C) $[=]V$.
R : electric resistance $[=]\Omega$.
L : inductance $[=]H$.
C : capacitance $[=]F$.

(f) Consider an electrical RLC circuit with an LC tank circuit in series as shown in the figure.

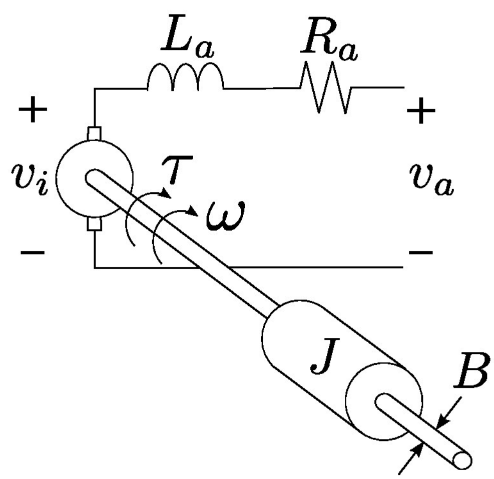


Where:

$u(t)$: source voltage $[=]V$.
$y(t)$: output voltage (voltage across R) $[=]V$.

R : electric resistance $[=]\Omega$.
L : inductance $[=]H$.
C : capacitance $[=]F$.

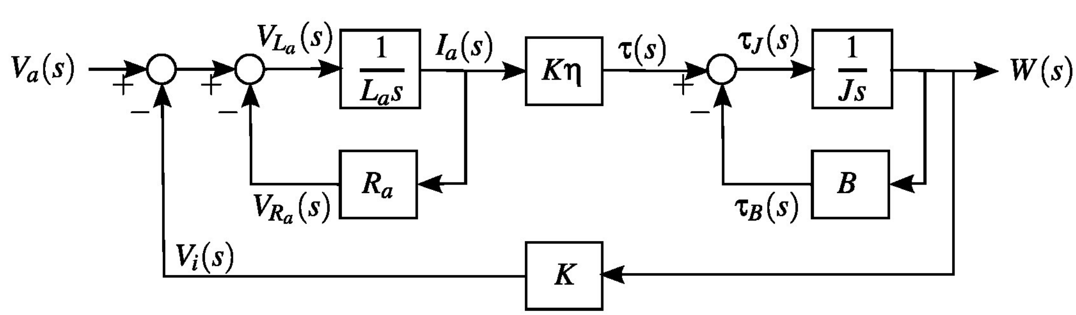
(g) Consider the DC motor with permanent magnet shown in the figure.



Where:

v_a : voltage across armature winding (input signal) $[=]V$.
i_a : current through armature winding $[=]A$.
R_a : armature winding resistance $[=]\Omega$.
L_a : armature winding inductance $[=]H$.
ω : angular velocity in motor axis (output signal) $[=]rad/s$.
J : moment of inertia of rotating parts of the motor $[=]kg.m^2$.
B : viscous friction coefficient of rotating parts of the motor $[=]kg.m^2/s$.
$v_i = K\omega$: voltage induced in the armature $[=]V$. K is a constant associated with the motor.
$\tau = K\eta i_a$: torque produced by the motor $[=]N \cdot m$. η is the efficiency of the motor $(0 \leq \eta \leq 1)$.

5. Consider the block diagram of a DC motor with permanent magnet shown in the figure.



Obtain the following transfer functions:

(a) $\dfrac{W(s)}{V_a(s)}$

(b) $\dfrac{V_{L_a}(s)}{V_a(s)}$

(c) $\dfrac{V_{R_a}(s)}{V_a(s)}$

(d) $\dfrac{I_a(s)}{V_a(s)}$

(e) $\dfrac{\tau(s)}{V_a(s)}$

(f) $\dfrac{\tau_J(s)}{V_a(s)}$

(g) $\dfrac{\tau_B(s)}{V_a(s)}$

(h) $\dfrac{V_i(s)}{V_a(s)}$

6. Get the equivalent transfer function of the systems with the block diagrams obtained in numeral 4.

7. Use the software tool Simulink® of Matlab® to draw a block diagram in which all the internal signals of the system can be seen for each of the systems in problem number 4. Connect a step to the system input and connect to each of the system signals a scope, including the output, so that the behavior of the system can be simulated for a specified time. Assign values to the different parameters of the model, such as $R = 1\,\Omega$, $L = 1\,H$, $C = 1\,F$. Verify that the simulation runs correctly.

8. For each of the following systems do the following.

   i. Draw a block diagram in terms of integrators, define the state variables, and obtain a representation in state space in controllable canonical form.

   ii. Draw a block diagram in terms of integrators, define the state variables, and obtain a representation in state space in observable canonical form.

   iii. Based on the controllable canonical form or the observable canonical form obtain a representation in in state space Jordan canonical form. Draw a block diagram in terms of integrators associated with this representation.

   (a) System modeled by the differential equation

$$\left(D^2 + D + 1\right) y(t) = x(t).$$

   (b) System modeled by the differential equation

$$\left(D^3 + 2D^2 + D + 1\right) y(t) = (D + 2)\, x(t).$$

(c) System modeled by the differential equation

$$\left(D^2 + 2D + 1\right) y(t) = x(t).$$

(d) System with transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{s^2 - 1}{s^2 + 4s + 3}.$$

(e) System with transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{2s - 3}{s^3 + 1}.$$

(f) System with transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{s}.$$

(g) System with transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{s + 4}{s^2 + 5s + 6}.$$