Automatic Flight Control

Classical Control Systems

Luis Benigno Gutiérrez Zea

Facultad de Ingeniería Aeronáutica
Universidad Pontificia Bolivariana

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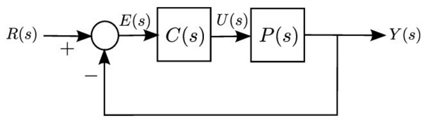
Classical control systems

- Classical control theory involves the design of control systems for single input single output (SISO) systems.
- The plant and the controller are assumed to be linear time invariant systems.
- In case of nonlinear plants the same type of linear controllers can be designed based on the linearized model of the plant.
- The linear control of nonlinear plants can be achieved when the plant presents smooth nonlinearities about the operating point.
- In the design process it is common to use frequency domain tools, but those can be combined with time domain simulation and optimization tools.

Notes

Classical control systems

- Revisit the control problem for a SISO linear time invariant plant with transfer function $P(s)$, using a unit feedback control structure as shown in the figure.



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Classical control systems

- The model of the plant, $P(s)$, is assumed to be known.
- For the classical control problem, the control system transfer function, $C(s)$, will be designed to fulfill some control requirements, in summary, to minimize the error $e(t)$ guaranteeing the robust stability and performance of the closed loop system.
- $C(s)$ is called the **controller** or **compensator**.

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Types of compensators

In classical control the compensators can be:

- **Proportional control** or **P control**.
- **Proportional-integral control** or **PI control**.
- **Proportional-derivative control** or **PD control**.
- **Proportional-integral-derivative control** or **PID control**.
- **Lead compensator**.
- **Lag compensator**.
- **Lead-lag compensator**.

Notes

P control

- In this case

$$C(s) = K_p \quad (1)$$

- K_p is the **proportional gain**.

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PI control

- In this case

$$C(s) = K_p + \frac{K_i}{s} \quad (2)$$

- K_p is the **proportional gain**.
- K_i is the **integral gain**.
- Sometimes the PI control transfer function is expressed as

$$C(s) = K_p\left(1 + \frac{1}{\tau_i s}\right) \quad (3)$$

- τ_i is the **integral time constant** or **reset**.
- Notice that $K_i = \frac{K_p}{\tau_i}$.

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PD control

- In this case

$$C(s) = K_p + K_d s \quad (4)$$

- K_p is the **proportional gain**.
- K_d is the **derivative gain**.
- Sometimes the PD control transfer function is expressed as

$$C(s) = K_p(1 + \tau_d s) \quad (5)$$

- τ_d is the **derivative time constant** or **rate**.
- Notice that $K_d = K_p \tau_d$.

Notes

PID control

- In this case

$$C(s) = K_p + \frac{K_i}{s} + K_d s \quad (6)$$

- K_p is the **proportional gain**.
- K_i is the **integral gain**.
- K_d is the **derivative gain**.
- Sometimes the PID control transfer function is expressed as

$$C(s) = K_p\left(1 + \frac{1}{\tau_i s} + \tau_d s\right) \quad (7)$$

- τ_i is the **integral time constant** or **reset**.
- τ_d is the **derivative time constant** or **rate**.
- Notice that $K_i = \frac{K_p}{\tau_i}$ and $K_d = K_p \tau_d$.

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Lag compensator

- In this case
$$C(s) = K_c \frac{\tau_g s + 1}{\alpha_g \tau_g s + 1} \quad (8)$$
- K_c is the gain of the lag compensator.
- τ_g is the lag compensator time constant.
- $\alpha_g > 1$ is a design parameter for the lag compensator.

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Lead compensator

- In this case
$$C(s) = K_c \frac{\tau_d s + 1}{\alpha_d \tau_d s + 1} \quad (9)$$
- K_c is the gain of the lead compensator.
- τ_d is the lead compensator time constant.
- $\alpha_d < 1$ is a design parameter for the lead compensator.

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Lead-lag compensator

- In this case
$$C(s) = K_c \frac{(\tau_d s + 1)}{(\alpha_d \tau_d s + 1)} \frac{(\tau_g s + 1)}{(\alpha_g \tau_g s + 1)} \quad (10)$$
- K_c is the gain of the lead-lag compensator.
- τ_d is the lead compensator time constant.
- $\alpha_d < 1$ is a design parameter for the lead compensator.
- τ_g is the lag compensator time constant.
- $\alpha_g > 1$ is a design parameter for the lag compensator.

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Design requirements in the time domain

The closed loop control system has some design requirements:

- Stability: the closed loop system should be stable.
- Performance:
  - Steady state response for a specified reference signal, $r(t)$:
    - $|e_{ss}| < e_{ss,max}$.
  - Transient state response referred to step response:
    - $M_p < M_{p,max}$
    - $t_r < t_{r,max}$
    - $t_s < t_{s,max}$
- Robustness: the closed loop system should behave well under internal and external uncertainties (model uncertainty, perturbations, and sensor noise).

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Design requirements in the frequency domain

The closed loop control system has some frequency domain design requirements:

- Stability: the closed loop system should be stable.
  - $PM > 0°$
  - $GM > 0dB$
- Performance:
  - $M_r < M_{r,max}$
  - w_c or bandwidth of control system.

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Stability margin requirements

The closed loop control system has stability margin requirements for a robust control system:

- Robust stability is guaranteed with good phase and gain margins.
  - $PM > PM_{min}$
  - $GM > GM_{min}$
- A good phase margin means $PM_{min} \geq 60°$.
- A good gain margin means $GP_{min} \geq 20dB$.

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Classical control system design

To design a classical control system:

- Select the control strategy.
- Tune the controller parameters. There are several methods:
  - Root locus method.
  - Frequency domain method.
  - Empirical tunning rules.
  - Optimization techniques.

Notes

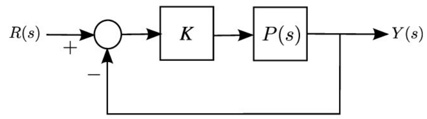
Root locus method

- The root locus method is based on a tool called the **root locus**.
- The **root locus** is a zero-pole diagram that shows the location of the closed loop poles in the complex plane parametrically respect to a design parameter.

Notes

Root locus

- Consider the closed loop system shown in the figure.



- Where $P(s) = \dfrac{N(s)}{D(s)}$ and K is a design gain.
- The closed loop transfer function is given by

$$\frac{Y(s)}{R(s)} = \frac{KP(s)}{1+KP(s)} = \frac{K\frac{N(s)}{D(s)}}{1+K\frac{N(s)}{D(s)}} = \frac{KN(s)}{D(s)+KN(s)} \tag{11}$$

Notes

Root locus

- The closed loop characteristic equation is

$$D(s) + KN(s) = 0 \tag{12}$$

- The **root locus** is the plot of the roots of (12) parametrically respect to K, for $0 < K < \infty$.
- The **complementary root locus** is the plot of the roots of (12) parametrically respect to K, for $-\infty < K < 0$.

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Root locus

- Notice that when $K \to 0$

$$\lim_{K\to 0} (D(s) + KN(s)) = D(s) = 0 \tag{13}$$

- Therefore, the root locus starts at the poles of the open loop system $P(s)$.
- Notice that when $K \to \infty$

$$\lim_{K\to \infty} \left(\frac{D(s)}{K} + N(s)\right) = N(s) = 0 \tag{14}$$

- Therefore, the root locus ends at the zeros of the open loop system.

Notes

Root locus

- The real axis to the left of an odd number of poles and zeros is part of the root locus.
- If $P(s)$ has m zeros and n poles with $m < n$, then there will be $n - m$ branches of the root locus that go to infinity.
- The $n - m$ asymptotes corresponding to those branches cross the real axis at α, with

$$\alpha = \frac{\sum_{i=1}^{n} p_i - \sum_{j=1}^{m} z_j}{n-m} \tag{15}$$

- where p_i with $i = 1, \ldots, n$ are the poles, and z_j with $j = 1, \ldots, m$ are the zeros of $P(s)$.
- The asymptotes form angles symmetrical respect to real axis. If $n - m$ is odd, one of the asymptotes will be the negative real axis.
- The breakaway points are located at the roots of $\dfrac{dP(s)}{ds} = 0$.

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Root locus

- In Matlab/Octave, the command **rlocus** allows to plot the root locus.
- For example, to plot the root locus of

$$P(s) = \frac{4}{s^3 + 5s^2 + 12s + 8}$$

- use the following code

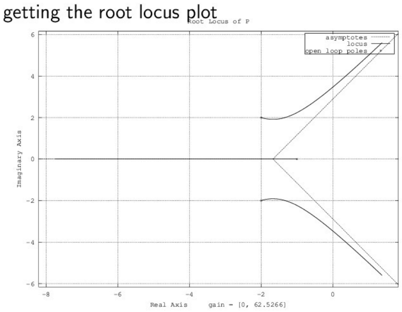
```
P = tf(4,[1 5 12 8])
rlocus(P); grid on
```

- or try the *sisotool* in Matlab.

Notes

Root locus

- getting the root locus plot



Notes

Design of controllers using the root locus

- To design a controller, the root locus can be used based on these guidelines

Controller type	Add	Tune
P Control	-	Gain
PD Control	One negative real zero	Gain - zero position
PI Control	One negative real zero and one pole at zero	Gain - zero position
PID Control	Two negative real zeros and one pole at zero	Gain - zero positions
Lead Control	One negative real zero and one more negative real pole	Gain - zero position - pole position
Lag Control	One negative real pole and more negative real zero	Gain - pole position - zero position
Lead-lag Control	Two negative real zeros and two negative real poles	Gain - zero positions - pole positions

- The integral part allows to obtain a zero steady state error for a step reference (in case the plant has no poles at zero).
- The derivative part should be used to stabilize the plant.

Notes

Frequency domain method

- Tune the gain of the controller or add a pole at zero (PI or PID controllers) to fulfill steady state requirements.
- Use the locations of poles and zeros of the compensator to shape the open loop transfer function trying to reach the desired gain and phase margins.
- Verify the time response of the closed loop system or repeat the process until a satisfactory design is found.

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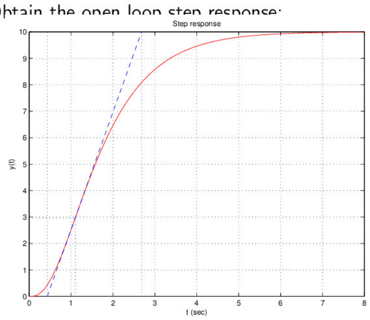
Empirical tunning rules

- Empirical tuning rules are methods for tuning PID controllers based on data taken from the open loop or closed loop response of the plant.
- These methods do not require to know the model of the process, they require access to the model to perform some tests previous to the tuning.
- There are many rules that have been devised for the purpose of tuning the controllers but the most famous are the **Ziegler-Nichols rules**.

Notes

Ziegler-Nichols open loop method

- Obtain the open loop step response:

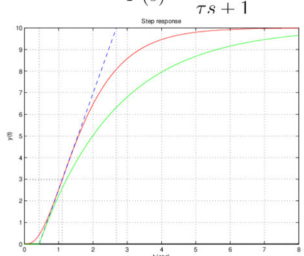


Notes

Ziegler-Nichols open loop method

- Measure the process parameters K, τ, t_d to approximate the process dynamics by a first order system with time delay:

$$P(s) = \frac{Ke^{-st_d}}{\tau s + 1}$$



Notes

Ziegler-Nichols open loop method

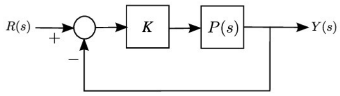
- Obtain the gains of the controller using

Controller type	K_p	τ_i	τ_d
P	$\frac{\tau}{t_d}$	∞	0
PI	$0.9\frac{\tau}{t_d}$	$\frac{t_d}{0.3}$	0
PID	$1.2\frac{\tau}{t_d}$	$2t_d$	$0.5t_d$

Notes

Ziegler-Nichols closed loop method

- Control the plant with a with a proportional control in a closed loop control structure:



- Find the step response of the closed loop system increasing the controller gain, K, until a critical value is found that makes the closed loop system to be marginally stable (i.e. the step response is a constant magnitude oscillatory response).
- The value of the gain obtained under this condition is the **critical gain**, K_{cr}.
- Record the period of the oscillation in the closed loop response, this is the **critical period**, T_{cr}.

Notes

Ziegler-Nichols closed loop method

- Obtain the gains of the controller using

Controller type	K_p	τ_i	τ_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{T_{cr}}{1.2}$	0
PID classical ZN rule	$0.6K_{cr}$	$\frac{T_{cr}}{2}$	$\frac{T_{cr}}{8}$
PID Pessen integral rule	$0.7K_{cr}$	$\frac{T_{cr}}{0.4}$	$0.15T_{cr}$
PID some overshoot	$0.33K_{cr}$	$\frac{T_{cr}}{2}$	$\frac{T_{cr}}{3}$
PID no overshoot	$0.2K_{cr}$	$\frac{T_{cr}}{2}$	$\frac{T_{cr}}{3}$

Notes

Optimization of controller gains

- If the model of the plant is known, some optimization techniques can be used to optimize the gains of the controllers.
- Define a performance index to minimize, for instance $\mathbf{ISECE} = \int_0^\infty |e(t)|^2 + \gamma\, |u(t)|^2 \, dt.$
- In practice impose constraints to the gains to guarantee a solution of the optimization problem, for instance $0 \le K_p < K_{p,max}$, $0 \le K_i < K_{i,max}$, $0 \le K_d < K_{d,max}$.
- Solve the optimization problem under given constraints to find the optimal values of the controller gains.

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Example 1

Use the root locus method to design a controller for the plant with transfer function

$$\frac{52.59}{s^3 + 23.92s^2 + 134.5s + 215.5} \quad (16)$$

In the design process try several kinds of controllers:

1. P control.
2. PD control.
3. PI control.
4. PID control.

In each case try to design the best control trying to fulfill the following requirements

$$e_{ss} < 1\%$$
$$M_p < 5\%$$
$$t_s < 1s$$

If it is not possible to fulfill all of the requirements explain why in each case.

Notes

Example 2

Use the Ziegler-Nichols method to design P, PI and PID controllers for the plant with transfer function

$$\frac{79.77}{s^3 + 26.5s^2 + 221.3s + 583.8} \quad (17)$$

Modify the obtained gains trying to fulfill the following requirements

$$e_{ss} < 1\%$$
$$M_p < 5\%$$
$$t_s < 1s$$

If it is not possible to fulfill all of the requirements explain why in each case.

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Example 3

Optimize the gains of the PID controllers designed in examples 1 and 2, finding the gains that minimize the performance index

$$\mathbf{ISECE} = \int_0^\infty |e(t)|^2 + \gamma\, |u(t)|^2 \, dt$$

with the restrictions

$$0 \le K_p < 100$$
$$0 \le K_i < 50$$
$$0 \le K_d < 20$$

En each case choose the value of γ trying to improve the response of the controller and avoiding excessive control effort.

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