Automatic Flight Control

Feedback Control Systems

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Control theory

- Control theory is a branch of mathematics and engineering that study the behavior of dynamical systems and through that knowledge study the way to modify their behavior in a desired way.
- According to the way the control theory problems are faced there are two broad divisions in control theory:

  Classical control theory. In many of the cases consider SISO systems, and is mostly based on frequency domain techniques.

  Modern control theory. Consider the general case of MIMO systems and is based on the state space representation of the systems.

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Problems in control theory

There are many problems in control theory, among them some of the more important are:

- System modeling.
- System identification.
- The control problem.
- The estimation problem.

Notes

System modeling problem

- Based on nature laws and physical principles model the system behavior in a two-step process:

  Physical modeling: Decide which nature laws and physical principles are going to be considered and which effects will not be taken into account, and the level of detail of the considerations. The result of this step is the **physical model**.

  Mathematical modeling: Represent the physical model by a set of mathematical equations, sufficient to establish a relationship among the variables under consideration. The result of this step is the **mathematical model**.

Notes

System identification problem

- Obtain a mathematical model which approximates the behavior of a system based on experimental data resulting from some tests on the physical system.
- The system identification is performed in different forms:
  - **System identification** per se, when the structure of the model is unknown and a black box model is obtained which approximates the behavior of the physical system in some interest domain.
  - **Parameter identification**, when the structure of the model is known, and a estimation of the unknown parameters is obtained based on some statistical treatment of the experimental data.

Notes

The control problem

- Given a dynamical system modeled by $y(t) = \mathscr{H}\left(u(t)\right)$, determine $u(t)$ such that the error, $e(t) = y_d(t) - y(t)$, is bounded by some specified value ϵ, that is $\|e(t)\| = \|y_d(t) - y(t)\| < \epsilon$ for $t \geq t_0$, where $y_d(t)$ is the desired value of the system output.
- In some cases it is also required that $\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|y_d(t) - y(t)\| = 0$.
- In some cases the error is measured respect to the state instead, so $e(t) = x_d(t) - x(t)$, where $x_d(t)$ is the desired value of the system state.

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The control problem

- In practice this problem must be solved knowing that the mathematical model of the system (model of $\mathscr{H}(\cdot)$) is not known exactly, this is called **uncertainty in the model**.
- To improve the **robustness** (tolerance to uncertainty) it is common to use feedback, that is $y(t)$ is measured and the knowledge of $y(t)$ is used to calculate $u(t)$.
- However, in practice the measure of $y(t)$ has some **uncertainty** due to sensor errors and noise. The controller must also cope with these errors.
- When the problem is solved, the system that implements the algorithm to calculate $u(t)$ is called the **control system**.

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The estimation problem

- Obtain an approximation (an estimate) of the state or a set of unknown system parameters based on some data taken from the system.
- The estimation problem is performed in different forms:
  - **State estimation**, when the the state of the system is estimated based on input/output information taken from the real system (state observation).
  - **Parameter estimation** when input/state/output information from the physical system is used to estimate a set of unknown parameters of the system (parameter identification).

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Issues in control theory

The most important issues to consider in control theory are:

- Stability.
- Controllability.
- Observability.
- Robustness.

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Some strategies for control systems

There are infinite solutions to the control problem. Some strategies to solve the control problem are:

- Classical control.
- Linear control.
- Nonlinear control.
- Optimal control.
- Robust control.
- Adaptive control.
- Intelligent control.

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Other topics in control theory

Other topics in control theory are:

- Discrete time systems and control (digital control).
- Discrete event systems.
- Hybrid systems.
- Stochastic systems and stochastic control.
- Process control.
- Control applications in engineering systems and sciences.

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Control system structure

The structure of the control system can be one of the following:

- Feedforward control.
- Feedback control.
- Feedback/Feedforward control.

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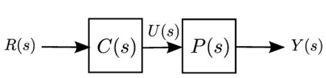
Feedforward control structure

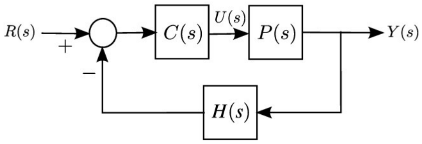


Where

- $r(t)$: reference, set point or desired value of the controlled variable.
- $y(t)$: controlled variable.
- $u(t)$: manipulated variable.
- $C(s)$: controller (control system) transfer function.
- $P(s)$: plant or process transfer function.

Notes

Feedback control structure



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- $C(s)$: controller (control system) transfer function.
- $P(s)$: plant or process transfer function.
- $H(s)$: sensor transfer function.

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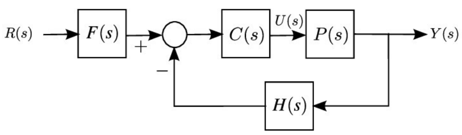
Feedback control structure with prefilter



Where

- $r(t)$: reference, set point or desired value of the controlled variable.
- $y(t)$: controlled variable.
- $u(t)$: manipulated variable.
- $F(s)$: prefilter transfer function.
- $C(s)$: controller (control system) transfer function.
- $P(s)$: plant or process transfer function.
- $H(s)$: sensor transfer function.

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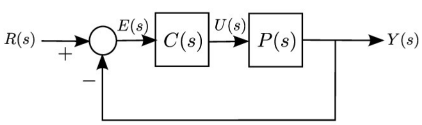
Unit feedback control structure



Where

- $r(t)$: reference, set point or desired value of the controlled variable.
- $y(t)$: controlled variable.
- $e(t)$: error, $e(t) = r(t) - y(t)$.
- $u(t)$: manipulated variable.
- $C(s)$: controller (control system) transfer function.
- $P(s)$: plant or process transfer function.

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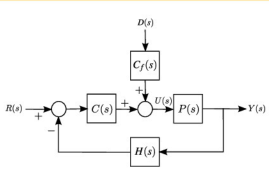
Feedback/Feedforward control structure

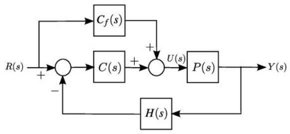


Where

- $r(t)$: reference, set point or desired value of the controlled variable.
- $y(t)$: controlled variable.
- $u(t)$: manipulated variable.
- $d(t)$: disturbance.
- $C_f(s)$: feedforward control system transfer function.
- $C(s)$: feedback control system transfer function.
- $P(s)$: plant or process transfer function.
- $H(s)$: sensor transfer function.

Notes

Feedback/Feedforward control structure



Where

- $r(t)$: reference, set point or desired value of the controlled variable.
- $y(t)$: controlled variable.
- $u(t)$: manipulated variable.
- $C_f(s)$: feedforward control system transfer function.
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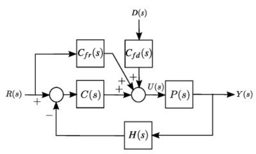
- $P(s)$: plant or process transfer function.
- $H(s)$: sensor transfer function.

Notes

Feedback/Feedforward control structure



Where

- $r(t)$: reference, set point or desired value of the controlled variable.
- $y(t)$: controlled variable.
- $u(t)$: manipulated variable.
- $d(t)$: disturbance.
- $C_{fr}(s)$: reference feedforward control system transfer function.
- $C_{fd}(s)$: disturbance feedforward control system transfer function.
- $C(s)$: feedback control system transfer function.
- $P(s)$: plant or process transfer function.
- $H(s)$: sensor transfer function.

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Feedback and/or Feedforward control structure?

- Feedforward control is not robust: it is not good to face uncertainties (disturbances or model errors).
- Feedforward control may not be feasible in some situations like the control of non-minimum phase systems.
- Feedforward control cannot stabilize an unstable plant.
- Feedforward control can be used alone in the control of simple and stable plants where the stability and robustness are not issues.
- Feedback control is more robust and can stabilize an unstable plant, thats why this is the preferred scheme.
- A good feedback control system can achieve good performance with robust stability in the closed loop system.
- Feedforward control can be used to complement a feedback control structure to improve the disturbance rejection when the disturbances are measurable.

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Control system design

To design a control system, follow these steps:

- Clearly state the control problem for the application of interest.
- Obtain the mathematical model of the plant or process to be controlled if required.
- Estimate the parameters of the plant or process to be controlled if required.
- Establish the control system requirements or specifications according to the application of interest.
- Select a control strategy according to the application of interest.
- Design the control system structure/parameters according to the selected control strategy and the plant or process to be controlled.

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Control system requirements or specifications

The control system must fulfill some requirements:

- Stability requirements or specifications.
- Performance requirements or specifications.
  - Steady state requirements or specifications.
  - Transient state requirements or specifications.
  - Frequency response requirements or specifications.
  - Sometimes, for the purpose of optimization, the requirements are summarized in a performance index.
- In practice nonlinearities of actuators and sensors and limits on some state variables must be taken into account in the design process.
- Robustness issues have to be considered also.

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Stability requirements

- The control system must be stable.
- The whole system must be stable (controller+plant).
- The stability criteria must be stated (we will use BIBO stability).
- A **level of stability** may be specified, i.e. **relative stability** specifying the possible locations of the closed loop poles.

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Performance requirements

Some of the performance requirements depend on the **error**, defined as

$$e(t) = r(t) - y(t) \tag{1}$$

Where $y(t)$ is the controlled variable and $r(t) = y_d(t)$ is the desired value of the controlled variable, the reference value or the set point. The performance requirements are specified as

- Steady state requirements.
- Transient state requirements.
- Frequency response requirements.
- Performance index optimality requirements.

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Steady state requirements

- The steady state error must be bounded by some specified value when the reference is a specified signal:

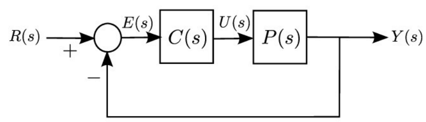
$$|e_{ss}| = \left|\lim_{t\to\infty} e(t)\right| < e_{ss,max} \tag{2}$$

for some specified reference signal $r(t)$.

Notes

Relationship between the type of the open loop transfer function and the steady state error

- Consider a unit feedback control system of the form



- The open loop transfer function will be $C(s)P(s)$, the transfer function of the loop when the feedback loop is broken.

Notes

Relationship between the type of the open loop transfer function and the steady state error

- Then, the steady state error can be calculated using the final value theorem of the Laplace transform as

$$\begin{aligned} e_{ss} &= \lim_{t\to\infty} e(t) \\ &= \lim_{s\to 0} sE(s) \\ &= \lim_{s\to 0} \frac{s}{1 + C(s)P(s)} R(s) \end{aligned}$$

for some specified reference signal $r(t)$, with $R(s) = \mathscr{L}\{r(t)\}$.

- The **type** of the open loop system is the number of poles in the origin $(s = 0)$ of the open loop transfer function, $C(s)P(s)$.

Notes

Relationship between the type of the open loop transfer function and the steady state error

- Consider an open loop transfer function of **type zero**.
- If the reference is a step signal, such that $R(s) = \frac{1}{s}$, then

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} \frac{1}{1 + C(s)P(s)} \\ &= \frac{1}{1 + \lim_{s\to 0} C(s)P(s)} \\ &= \frac{1}{1 + K_p} \end{aligned}$$

where $K_p = \lim_{s\to 0} C(s)P(s)$ is the **position error constant** of the system.

- For an open loop transfer function of **type zero**, $e_{ss} = \infty$ if the reference signal is a ramp, a parabola, a cubic, . . .

Notes

Relationship between the type of the open loop transfer function and the steady state error

- Consider an open loop transfer function of **type one**.
- If the reference is a ramp signal, such that $R(s) = \frac{1}{s^2}$, then

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} \frac{1}{s\,(1 + C(s)P(s))} \\ &= \frac{1}{\lim_{s\to 0} sC(s)P(s)} \\ &= \frac{1}{K_v} \end{aligned}$$

where $K_v = \lim_{s\to 0} sC(s)P(s)$ is the **velocity error constant** of the system.

- For an open loop transfer function of **type one**, $e_{ss} = 0$ if the reference signal is a step and $e_{ss} = \infty$ if the reference signal is a parabola, a cubic, ...

Notes

Relationship between the type of the open loop transfer function and the steady state error

- Consider an open loop transfer function of **type two**.
- If the reference is a parabola signal, such that $R(s) = \frac{1}{s^3}$, then

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} \frac{1}{s^2\,(1 + C(s)P(s))} \\ &= \frac{1}{\lim_{s\to 0} s^2C(s)P(s)} \\ &= \frac{1}{K_a} \end{aligned}$$

where $K_a = \lim_{s\to 0} s^2C(s)P(s)$ is the **acceleration error constant** of the system.

- For an open loop transfer function of **type two**, $e_{ss} = 0$ if the reference signal is a step or a ramp, and $e_{ss} = \infty$ if the reference signal is a cubic, ...

Notes

Relationship between the type of the open loop transfer function and the steady state error

In summary

e_{ss}

	Reference signal			
Type of $C(s)P(s)$	Step	Ramp	Parabola	Cubic
0	$\frac{1}{1+K_p}$	∞	∞	∞
1	0	$\frac{1}{K_v}$	∞	∞
2	0	0	$\frac{1}{K_a}$	∞

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Transient state requirements

- The transient behavior of the controller+plant must follow some transient response requirements: it is usual to specify the transient requirements in terms of the step response (the response of the controller+plant when the reference signal is a step signal).
- The requirements are specified in terms of
  - M_p : Overshoot.
  - t_r : Rise time.
  - t_s : Settling time.

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Transient state requirements

Where

$$M_p = \frac{y_{max} - y_{ss}}{y_{ss}} < M_{p,max} \quad (3)$$
$$t_r = t_2 - t_1 < t_{r,max} \quad (4)$$
$$t_s < t_{s,max} \quad (5)$$

where $y(t_1) = 0.1 * y_{ss}$, $y(t_2) = 0.9 * y_{ss}$ with $y_{ss} = \lim_{t \to \infty} y(t)$, and t_s is the minimum value of t_s such that $|y(t) - y_{ss}| < 0.02 y_{ss}$ for $t \geq t_s$.

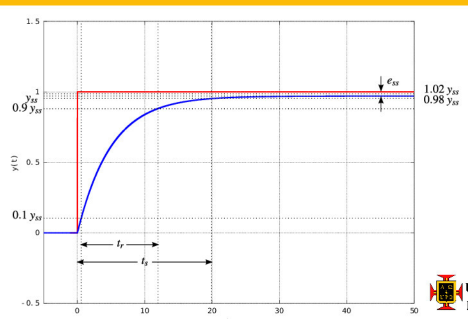
Notes

Example: transient state parameters for a first order system

- When the closed loop transfer function is of first order such that
$$\frac{Y(s)}{R(s)} = \frac{K}{\tau s + 1}$$
- In this case the step response of the closed loop system will be
$$y(t) = K\left(1 - e^{-t/\tau}\right) 1(t)$$

Notes

Example: transient state parameters for a first order system



Notes

Example: transient state parameters for a first order system

Then,

$$M_p = 0$$
$$t_r = (ln(0.9) - ln(0.1))\,\tau \approx 2.1972\tau$$
$$t_s = -\,ln(0.02)\tau \approx 4\tau$$

Notes

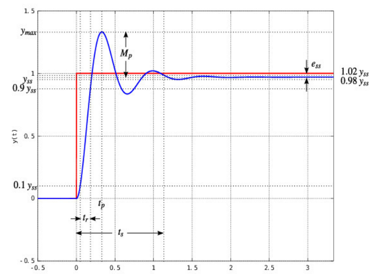
Example: transient state parameters for a second order system

- When the closed loop transfer function is of second order such that
$$\frac{Y(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$
- For the underdamped case, with $0 < \zeta < 1$, the step response of the closed loop system will be

$$y(t) = K\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta w_n t} Sen\left(w_n\sqrt{1-\zeta^2}\,t + Tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)\right] 1(t)$$

Notes

Example: transient state parameters for a second order system



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Example: transient state parameters for a second order system

Then,

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$
$$t_p = \frac{\pi}{\sqrt{1-\zeta^2}\omega_n}$$
$$t_r \approx \frac{0.8\,(\pi - cos^{-1}(\zeta))}{\sqrt{1-\zeta^2}\omega_n}$$
$$t_s \approx -\frac{ln(0.02)}{\zeta\omega_n} \approx \frac{4}{\zeta\omega_n}$$

Notes

Frequency response requirements

Closed loop frequency response characteristics:

Resonance frequency: $\omega_r = \underset{\omega}{\operatorname{argmax}} \left|\frac{Y(j\omega)}{R(j\omega)}\right|$.

Resonance peak: $M_r = \max_{\omega} \left|\frac{Y(j\omega)}{R(j\omega)}\right|$.

Bandwidth or cutoff frequency: ω_c such that $\left|\frac{Y(j\omega)}{R(j\omega)}\right| dB < \left|\frac{Y(0)}{R(0)}\right| dB - 3dB$ for $\omega > \omega_c$

Rolloff: Slope of $\left|\frac{Y(j\omega)}{R(j\omega)}\right| dB$ for $\omega = \omega_c$.

Notes

Example: closed loop frequency response characteristics for a first order system

- When the closed loop transfer function is of first order such that
$$\frac{Y(s)}{R(s)} = \frac{1}{\tau s + 1}$$
- In this case the frequency response of the closed loop system will be
$$\frac{Y(j\omega)}{R(j\omega)} = \frac{1}{1 + j\omega\tau}$$

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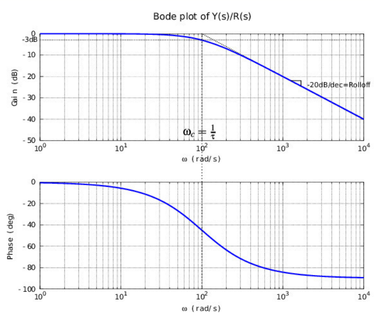
Example: closed loop frequency response characteristics for a first order system



Notes

Example: closed loop frequency response characteristics for a first order system

Then,

$$\begin{aligned} \omega_r &= 0 \\ M_r &= 0dB \\ \omega_c &= \frac{1}{\tau} rad/s \\ Rolloff &= -20dB/dec \end{aligned}$$

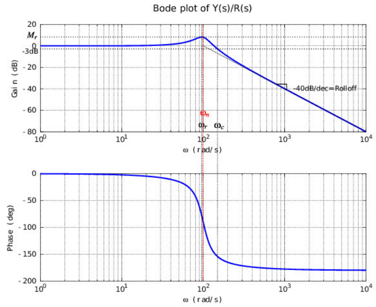
Notes

Example: closed loop frequency response characteristics for a second order system

- When the closed loop transfer function is of second order such that
$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$
- In this case the frequency response of the closed loop system will be
$$\frac{Y(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega}$$

Notes

Example: closed loop frequency response characteristics for a second order system



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Example: closed loop frequency response characteristics for a second order system

Then,

$$\omega_r = \begin{cases} \sqrt{1-2\zeta^2}\omega_n, & \zeta < \frac{\sqrt{2}}{2} \\ 0rad/s, & \zeta \geq \frac{\sqrt{2}}{2} \end{cases}$$

$$M_r = \begin{cases} -20 \cdot log\left(2\zeta\sqrt{1-\zeta^2}\right) dB, & \zeta < \frac{\sqrt{2}}{2} \\ 0dB, & \zeta \geq \frac{\sqrt{2}}{2} \end{cases}$$

$$\omega_c \begin{cases} > \omega_n, & \zeta < \frac{\sqrt{2}}{2} \\ = \omega_n, & \zeta = \frac{\sqrt{2}}{2} \\ < \omega_n, & \zeta > \frac{\sqrt{2}}{2} \end{cases}$$

$$Rolloff = -40dB/dec$$

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Performance indices

Some performance indices are commonly used in control system design problems:

- $\mathbf{ISE} = \int_0^\infty |e(t)|^2 \, dt$: Integral of square error.
- $\mathbf{IAE} = \int_0^\infty |e(t)| \, dt$: Integral of absolute error.
- $\mathbf{ITSE} = \int_0^\infty t\,|e(t)|^2 \, dt$: Integral of time times square error.
- $\mathbf{ITAE} = \int_0^\infty t\,|e(t)| \, dt$: Integral of time times absolute error.

In practice, a performance index can include a part related to the manipulated variable, to optimize also the control effort or the energy used by the controller.

For example $\mathbf{ISECE} = \int_0^\infty |e(t)|^2 + \gamma\,|u(t)|^2 \, dt$

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Stability margin requirements

Gain and Phase margins:

Phase margin: $PM = 180° + \angle G(j\omega_1)H(j\omega_1)$ where $|G(j\omega_1)H(j\omega_1)| = 1 \ (0dB)$.

Gain margin: $GM \ dB = -20Log\,|G(j\omega_2)H(j\omega_2)| \ dB$ where $\angle G(j\omega_2)H(j\omega_2) = -180°$.

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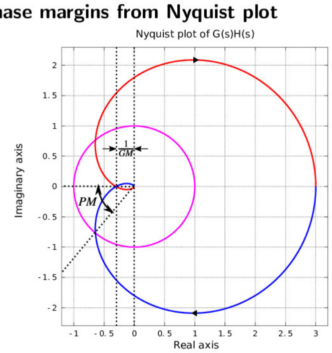
Stability margin requirements

Gain and Phase margins from Nyquist plot



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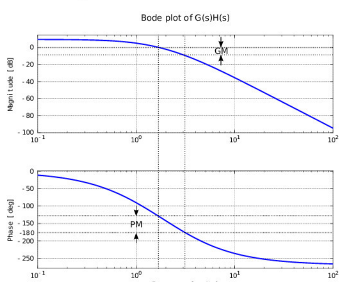
Stability margin requirements

Gain and Phase margins from Bode plot



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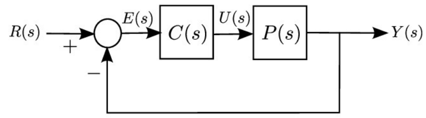
Examples

Consider the closed loop control system shown in the figure



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Examples

Use analytical and/or computational tools to determine the conditions for stability, calculate the steady state error, transient state parameters, frequency response parameters and performance indices for each of the following cases.

1. Proportional control (**control P**) on a first order plant: $P(s) = \dfrac{K}{\tau s + 1}$ and $C(s) = K_p$
2. Proportional control (**control P**) on a second order plant: $P(s) = \dfrac{K}{s(\tau s + 1)}$ and $C(s) = K_p$
3. Proportional-integral control (**control PI**) on a first order plant: $P(s) = \dfrac{K}{\tau s + 1}$ and $C(s) = K_p + \dfrac{K_i}{s}$
4. Proportional-derivative control (**control PD**) on a second order plant: $P(s) = \dfrac{K}{s(\tau s + 1)}$ and $C(s) = K_p + K_d s$
5. Proportional-integral control (**control PI**) on a second order plant: $P(s) = \dfrac{K}{s(\tau s + 1)}$ and $C(s) = K_p + \dfrac{K_i}{s}$

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