Automatic Flight Control

Basic Concepts

Luis Benigno Gutiérrez Zea

Facultad de Ingeniería Aeronáutica
Universidad Pontificia Bolivariana

First semester - 2025

Notes

Outline

Notes

Outline

Notes

Signals

- A **signal** is a representation of the behavior or attributes of some phenomenon.
- A signal is represented mathematically by a function:
- Let T be the set of possible times, and $t \in T$ be the time.
- Then, a signal f can be represented by function $f : T \to \vartheta$, so $f(t) \in \vartheta$ represents the value of the signal at time $t \in T$.
  - The independent variable is the time.
  - The dependent variables is the variable representing some attribute of the phenomenon, it is usually a **physical variable**.

Notes

Signals

Signals can be represented as any mathematical function by:

- a data table specifying the value of the signal or each time value or,
- a mathematical expression that allows to calculate the value of the signal or each time value or,
- a plot showing the value of the signal for each time value.

Notes

Classification of signals

- Depending on the continuous or discrete nature of the sets T and ϑ, the signals can be classified as:
  - **Discrete time signals** when T is a discrete set.
  - **Continuous time signals** when T is a continuous set.
  - **Discrete variable signals** when ϑ is a discrete set.
  - **Continuous variable signals** when ϑ is continuous set.
  - A signal is **digital** when it is discrete time and discrete variable.
  - A signal is **analog** when it is continuous time and continuous variable.

Note:

- A continuous set is a set for which there exists a bijective function with the real set, $\mathbb{R}$.
- A discrete set is a set for which there exists a bijective function with a nonempty subset of the integers set, $\mathbb{Z}$.

Notes

Classification of signals

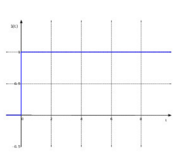
- A signal is a **deterministic signal** if the signal is known with complete certainty.
- A signal is a **random signal** or **stochastic signal** if there is uncertainty in any feature of the signal.
- A signal $x(t)$ is **periodic** if there exists $T > 0$ such that $x(t) = x(t + T)$ for all $t \in \mathbb{R}$. Otherwise the signal is **aperiodic**.

Notes

Some simple signals: the singular signals

- Unit step signal:

$$1(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

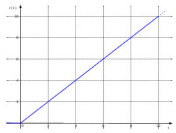


Notes

Some simple signals: the singular signals

- Unit ramp signal:

$$r(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$

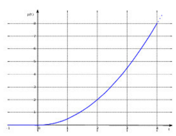


Notes

Some simple signals: the singular signals

- Unit parabola signal:

$$p(t) = \begin{cases} 0, & t < 0 \\ \frac{t^2}{2}, & t \geq 0 \end{cases}$$

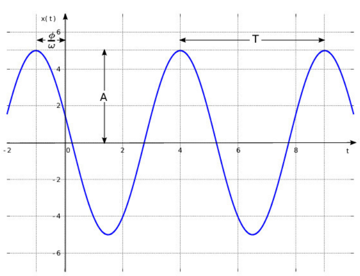


Notes

Some simple signals

- Sinusoidal signal.

$$x(t) = Acos(\omega t + \phi)$$



Notes

Sinusoidal signal parameters

- Sinusoidal signal.

$$x(t) = Acos(\omega t + \phi)$$

A: amplitude

ω: angular frequency $[=] rad/s$

ϕ : phase $[=] rad$

$f = \frac{\omega}{2\pi}$: frequency $[=] Hz = s^{-1}$

$T = \frac{1}{f} = \frac{2\pi}{\omega}$: period $[=] s$

Notes

Operations with signals

- Addition/subtraction.
- Differentiation.
- Integration.

Notes

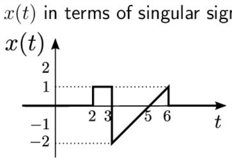
Representation of some signals in terms of singular signals

Example

Plot the signals

- $f(t) = 1(t+3) + r(t+2) - 2r(t) + r(t-2) - 1(t-3)$
- The **rectangular or unit pulse signal**: $\Pi(t) = 1(t+1/2) - 1(t-1/2)$
- The **triangle signal**: $\Lambda(t) = r(t+1) - 2r(t) + r(t-1)$

Represent the signal $x(t)$ in terms of singular signals



Notes

Outline

Notes

Systems

- A **system** is a set of interacting or interdependent components forming an integrated whole for a common purpose.
- A **subsystem** is a system that is part of another system.
- Signals are used for the variables that represent the behavior of the system.
- The system interacts with the environment through input and output signals.
- **Input signals** represent interactions from the environment acting on the system.
- **Output signals** represent interactions from the system acting on the environment.
- **Internal signals** represent interactions among internal components of the system.

Notes

Mathematical models

- A system can be represented by a **mathematical model**.
- A mathematical model is a mathematical representation of the system through a set of relations among the signals that represent the physical variables involved in the system.
- In the mathematical model also appear some **parameters** representing the properties or attributes of the components of the system.
- The solutions of the mathematical model approximate the behavior of the system.
- A system can be represented by many mathematical models with different levels of approximation of the system behavior. That's why there is no unique mathematical model for a system.

Notes

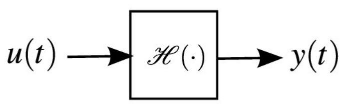
Mathematical model abstraction

- A system can be abstracted mathematically as an operator that maps the input signals into the output signals.
- That is:

$$y(t) = \mathcal{H}\{u(t)\}$$

where $u(t)$ is the input signal and $y(t)$ is the output signal.

- Graphically:



Notes

Classification of Mathematical models

- According to the number of inputs/outputs:
  - Single Input Single Output (SISO) systems.
    One input/one output.
  - Multiple Input Multiple Output (MIMO) systems.
    Several inputs and/or several outputs.
- According to the time:
  - Continuous time systems.
    System signals are continuous time.
  - Discrete time systems.
    System signals are discrete time.
- According to the dependent variable:
  - Continuous variable systems.
    System signals are continuous variable signals.
  - Discrete variable systems.
    System signals are discrete variable signals.

Notes

Classification of Mathematical models

- According to the determinism:
  - Deterministic systems.
    There is no uncertainty in system behavior.
  - Stochastic systems.
    There is uncertainty in system behavior.
- According to the distribution in space:
  - Lumped parameter systems.
    Position in the system is not relevant for the signals.
    Signals only depend on time.
  - Distributed parameter systems.
    Position in the system is not relevant for the signals.
    Signals are dependent on time and position.

Notes

Classification of Mathematical models

- According to the dynamism:
  - Instantaneous systems.
    A system is instantaneous if $y(t_0) = \mathcal{H}\{u(t_0)\}$ for all $t_0 \in \mathbb{R}$.
    That is, a system is instantaneous if $y(t_0)$ only depends on $u(t_0)$ for any input signal.
  - Dynamic systems.
    A system is dynamic if $y(t_0) = \mathcal{H}\{u(t)\}$ for some $t \neq t_0$, for some $t_0 \in \mathbb{R}$.
    That is, a system is dynamic if $y(t_0)$ may depend on $u(t)$ for some $t \neq t_0$.

Notes

Classification of Mathematical models

- According to the causality:
  - Causal systems.
    For a system such that $y_1(t) = \mathcal{H}\{u_1(t)\}$ and $y_2(t) = \mathcal{H}\{u_2(t)\}$, the system is causal if $y_1(t_0) = y_2(t_0)$ implies that $u_1(t) = u_2(t)$ for $t \leq t_0$.
    That is, a system is causal if the output a time t_0, $y(t_0)$, depends only on values of $u(t)$ for $t \leq t_0$ for all $t_0 \in \mathbb{R}$.
  - Non causal systems.
    A system is non causal if the output a time t_0, $y(t_0)$, may depend on values of $u(t)$ for some $t > t_0$ for some $t_0 \in \mathbb{R}$.

Notes

Classification of Mathematical models

- According to the variability in time:
  - Time invariant systems.
    A system is said to be time invariant if $y(t) = \mathcal{H}\{u(t)\}$ implies that $y(t - t_0) = \mathcal{H}\{u(t - t_0)\}$ for all $u(t)$ and for all $t_0 \in \mathbb{R}$.
  - Time variable systems.
    A system if said to be time variable if it is not a time invariant system.

Notes

Classification of Mathematical models

- According to the linearity:
  - Linear systems.
    A system is said to be linear if given that $y_1(t) = \mathcal{H}\{u_1(t)\}$ and $y_2(t) = \mathcal{H}\{u_2(t)\}$, that implies that $\mathcal{H}\{a_1u_1(t) + a_2u_2(t)\} = a_1\mathcal{H}\{u_1(t)\} + a_2\mathcal{H}\{u_2(t)\} = a_1y_1(t) + a_2y_2(t)$ for any $a_1, a_2 \in \mathbb{R}$, and any $u_1(t)$, $u_2(t)$.
  - Nonlinear systems.
    A system is said to be nonlinear if it is not a linear system.

Notes